Monte Carlo modeling of radiative transfer in snow

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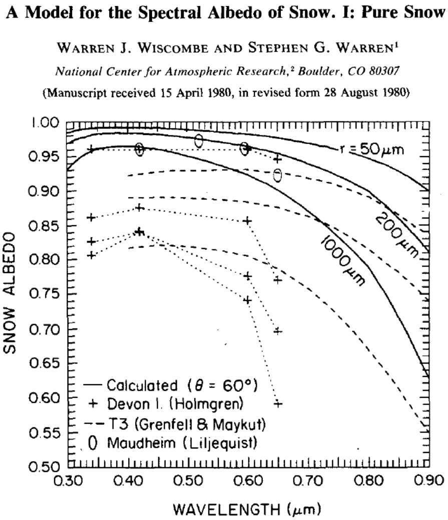
Modeling snow reflectance: Flashback to 1980



- Mie Theory applied to derive optical properties of ice particles and impurities
- Multiple-scattering approximation (delta-Eddington) to represent radiative transfer in the bulk medium
- Pure-snow model over-predicted reflectance in the visible spectrum
- Companion study incorporated impurities into model

What does snow albedo depend on?

- Snow grain size
- Solar zenith angle
- Content of absorbing impurities like black carbon, dust, algae, ash
- Snow thickness and underlying ground albedo

More nuanced definitions of albedo:

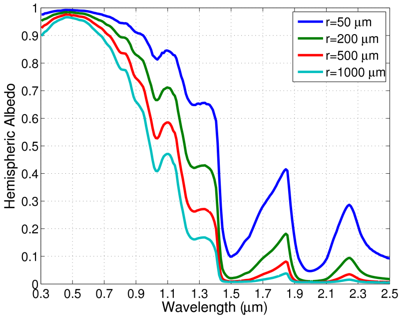
- *Spectral reflectance*: Wavelength- (or frequency-) dependent reflectance
- *Broadband albedo*: Integrated reflectance over a spectral range $(\lambda_1 - \lambda_2)$. Depends on spectral distribution of incident light:

$$A_{bb} = \frac{\int_{\lambda_1}^{\lambda_2} r_\lambda \, F_\lambda^\downarrow \, d\lambda}{\int_{\lambda_1}^{\lambda_2} F_\lambda^\downarrow \, d\lambda} = \frac{F_{bb}^\uparrow}{F_{bb}^\downarrow} \tag{1}$$

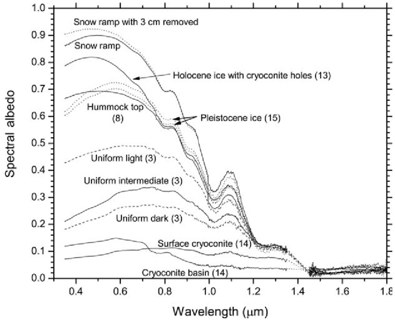
- *Directional reflectance*: Reflectance into a given direction, given a direction of incidence
- *Hemispheric albedo*: Hemispherically-integrated reflectance
- *Hemispheric broadband albedo*: ??

The color of snow and ice

... or spectral hemispheric reflectance



(a) Large variability in near-infrared albedo of snow with grain size



(b) Measured spectral reflectance of different surfaces on Greenland (Bøggild et al, 2010)

Basic ingredients for modeling snow radiative transfer

1. Knowledge of how likely the element is to interact with (i.e., scatter or absorb) incident radiation: *mass extinction coefficient* (k_{ext})
2. Knowledge of the relative likelihood of an extinction event being scattering or absorption: *single-scatter albedo*:

$$\tilde{\omega} = \frac{k_{\text{sca}}}{k_{\text{ext}}} \tag{2}$$

3. Mathematical description of how the element (snow grains) scatters, or re-directs radiation into different directions: *scattering phase function*
4. The mass or volume density of the element (snow grains)
5. A model of radiative transfer that accommodates *multiple scattering*
6. For inclusion of impurities like black carbon and dust, we also need to know properties 1–4 for these "elements"

Building an analytical RT model

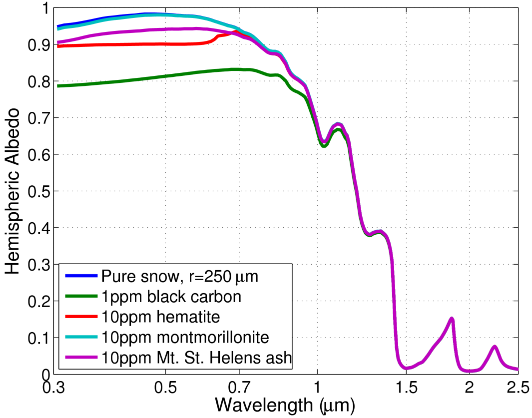
Complicated!

One solution that accommodates multiple scattering in snow

$$
\begin{aligned}
Qa_d = {} & 2P\left[(1-\gamma+\tilde{\omega}^*b^*)(1-\tau_0^*)-\frac{\gamma\tilde{\omega}^*(1+b^*)}{1-\tilde{\omega}^*}\right]\exp(-\tau_0^*) - 2P\left[\tilde{\omega}^*(1+b^*)\left(\frac{2}{\xi^2}+\frac{\gamma\tau_0^*}{1-\tilde{\omega}^*}\right)\right. \\
& \left. + (1-\gamma+\tilde{\omega}^*b^*)\tau_0^{*2}\right]\mathrm{Ei}(-\tau_0^*) + \frac{2\tilde{\omega}^*(1+b^*)}{\xi^2}\left[Q^+\{\mathrm{Ei}[-(1+\xi)\tau_0^*]+\xi-\ln(1+\xi)\}\right. \\
& \left. - Q^-\{\mathrm{Ei}[-(1-\xi)\tau_0^*]-\xi-\ln|1-\xi|\}\right] - \tilde{\omega}^*b^*(Q^+-Q^-),
\end{aligned}
$$

- (*Wiscombe and Warren*, 1980): Hemispheric albedo of a snowpack of finite thickness at a single wavelength for diffuse incident light (e.g., under a thick cloud)
- Again, the challenge is accounting for *multiple scattering*

Another analytical model: SNICAR



- Simulate it yourself at: http://snow.engin.umich.edu

Monte Carlo modeling

- Analytical approximations are possible when numerous assumptions are made
- An alternative approach is with *Monte Carlo* modeling: repeated random sampling from probability density functions to obtain statistical representation of the system's behavior
- This technique is applied in numerous fields (not just radiative transfer modeling)

Monte Carlo modeling

Pros:

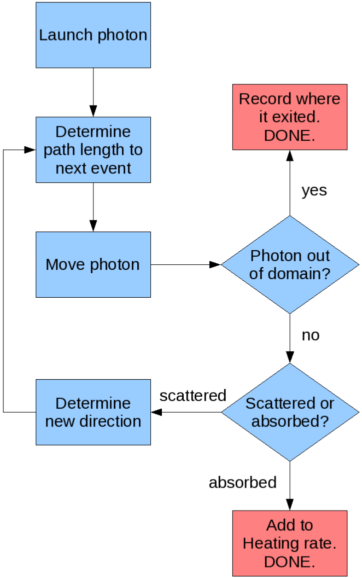
- Given enough sampling, the technique is highly accurate. (Accuracy limited much more by uncertainty in PDFs than in computational technique)
- It is robust: 3-D modeling and heterogeneous geometries can be more easily incorporated. Not restricted to plane-parallel assumptions
- Considering radiative transfer from the discrete photon perspective, this approach is physically-based and intuitive
- Relatively easy to program

Monte Carlo modeling

Cons:

- Computationally expensive – impractically so in many cases. Need to simulate many photons to obtain reliable results, and the number needed can become unreasonably large, especially with optically-thick media.
- Photons can become "lost" deep inside optically thick media like clouds and snow, requiring excessive computational time to resolve
- Cannot capture anywhere near the true number of photons! Rather, we hope to capture a *representative sample*.

Monte Carlo “decision tree” for modeling photons



Scattering phase function

- Again, we can think of p (or $p/4\pi$) as a probability density function. It must integrate to 1. (If a photon is scattered, it must go *somewhere*. And, we cannot end up with more photons, through scattering, than we started with).
- What controls p?
  - Particle size
  - Particle shape
  - Refractive index
- We can greatly simplify when particles are spheres (or when we assume particles are spheres). In this case, the geometry is *isotropic*, and we only need to consider the *scattering phase angle*: Θ (instead of four directions in the general case):

$$\cos\Theta = \hat{\mathbf{\Omega}}' \cdot \hat{\mathbf{\Omega}} \tag{3}$$

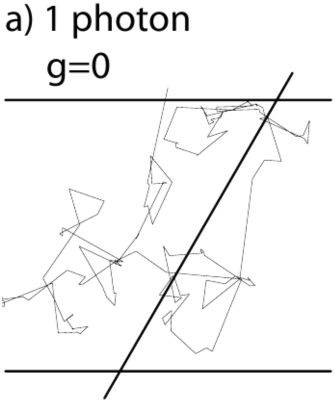
- Θ is simply the angle between the incident and scattered photon

Isotropic scattering

- Simplest scattering case: *isotropic scattering*: equal scattering in all directions. No information about scattering direction from incident direction.

$$p(\cos\Theta) = \text{constant} \tag{4}$$

- Isotropic scattering example inside of a cloud: "Aimless wandering".



Photon outcomes

Four possibilities for a photon incident on top of snowpack:

1. Passes through snow without being scattered once, or absorbed. The fraction that experiences this is *direct transmittance* (t_{dir}).
2. Scattered one or more times and then emerge from the *bottom* of the snowpack (where it is likely to be absorbed by ground). This fraction is the *diffuse transmittance* (t_{dif}).
3. Scattered one or more times and then emerge from the *top* of the snow. This fraction represents the *reflectance* (or albedo).
4. It can be absorbed (by ice, impurities, or air). This fraction is the *absorptance*.
5. The four fractions (probabilities) must sum to 1 for conservation:

$$t_{\text{dir}} + t_{\text{dif}} + r + a = 1 \tag{5}$$

The asymmetry parameter

- Scattering by real particles (snow grains) is never isotropic
- To accurately model scattering, we might need a complicated function to describe $p(\hat{\boldsymbol{\Omega}}', \hat{\boldsymbol{\Omega}})$.
- **For spheres, this function can be computed analytically with** *Mie Theory*
- But in many cases, we may only be concerned with *flux* (not intensity), in which case it is sufficient to know the relative proportion of photons scattered in the forward and backward hemispheres (i.e., the *backscatter fraction*), or the mean scattering angle.
- The *asymmetry parameter* (g) accomplishes this, describing the **average value of** $\cos\Theta$ for a large number of scattered photons:

$$g = \frac{1}{4\pi}\int_{4\pi} p(\cos\Theta)\cos\Theta d\omega \tag{6}$$

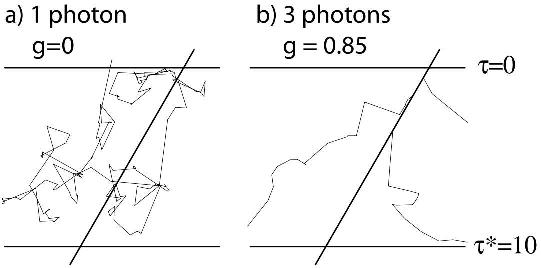
The asymmetry parameter

- Range of asymmetry parameter:

$$-1 \leq g \leq 1 \tag{7}$$

- If $g > 0$, photons are preferentially scattered into the forward hemisphere
- If $g < 0$, photons are preferentially scattered into the backward hemisphere
- In the case of isotropic scattering, what is g?
- What would $g = 1$ imply?
- Can $g = 0$ for non-isotropic scattering?
- **Aerosols, snow grains, and cloud droplets that scatter visible radiation typically have:** $0.8 \leq g \leq 0.9$**, meaning they are strongly** *forward-scattering*

Scattering and the asymmetry parameter



- *Isotropic* scattering on left, *forward-scattering* on right
- Cloud/snow optical depth is the same in both cases
- In $g = 0.85$ case, photons are much less likely to undergo rapid redirection
- Which case would exhibit greater *diffuse transmittance*? And greater albedo?

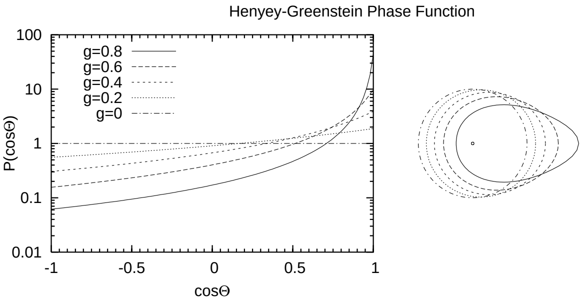
Henyey–Greenstein phase function

- Often g is sufficient to describe scattering direction, but sometimes we need more detail. The Henyey–Greenstein function "fills-in" a full scattering phase function, using only g:

$$p_{\mathrm{HG}}(\cos\Theta) = \frac{1 - g^2}{(1 + g^2 - 2g\cos\Theta)^{3/2}} \tag{8}$$

- Convenient and reasonably accurate in many cases, but it is an example of *downscaling* (creating higher resolution data from low-resolution input)

Henyey–Greenstein phase function



- For positive g, the function peaks increasingly in the forward direction (as we would hope), but remains smooth. Thus, it works fairly well for particles with large *size parameters* (snow grains, cloud droplets and aerosols)

Monte Carlo model demonstration