Detection and attribution of long-term change ("trend" detection)

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CSI, 19 July 2017

Photo: F. Zwiers (Strait of Juan de Fuca)

Introduction

- Today's talk
  - Detection and attribution of change in the mean state
  - Science using this technique has become the foundation for the series of attribution assessments that have been made by the IPCC
    - SAR – 1995 – "discernable evidence"
    - AR5 – 2013 – it is "extremely likely that most …"
- Tomorrow's talk
  - Detection and attribution of changes in extremes
  - Event attribution
- Key reference
  - WCRP summer school on extremes, ICTP, July, 2014

Outline

- Overview
- A simpler method
- A more complex method
- Covariance matrix estimation
- Discussion

Photo: F. Zwiers **(Marsh Wren)**

Definition of D & A

- *Detection* of change is defined as the process of demonstrating that climate or a system affected by climate has changed in some defined statistical sense without providing a reason for that change.

- *Attribution* is defined as the process of evaluating the relative contributions of multiple causal factors to a change or event with an assignment of statistical confidence.

- In WG1, casual factors usually refer to *external influences*, which may be *anthropogenic* (GHGs, aerosols, ozone precursors, land use) and/or *natural* (volcanic eruptions, solar cycle modulations).

IPCC Good Practice Guidance Paper on Detection and Attribution, 2010

Methods

- Involve simple statistical models
- Complex implementation due to data volumes (which are both small and large)

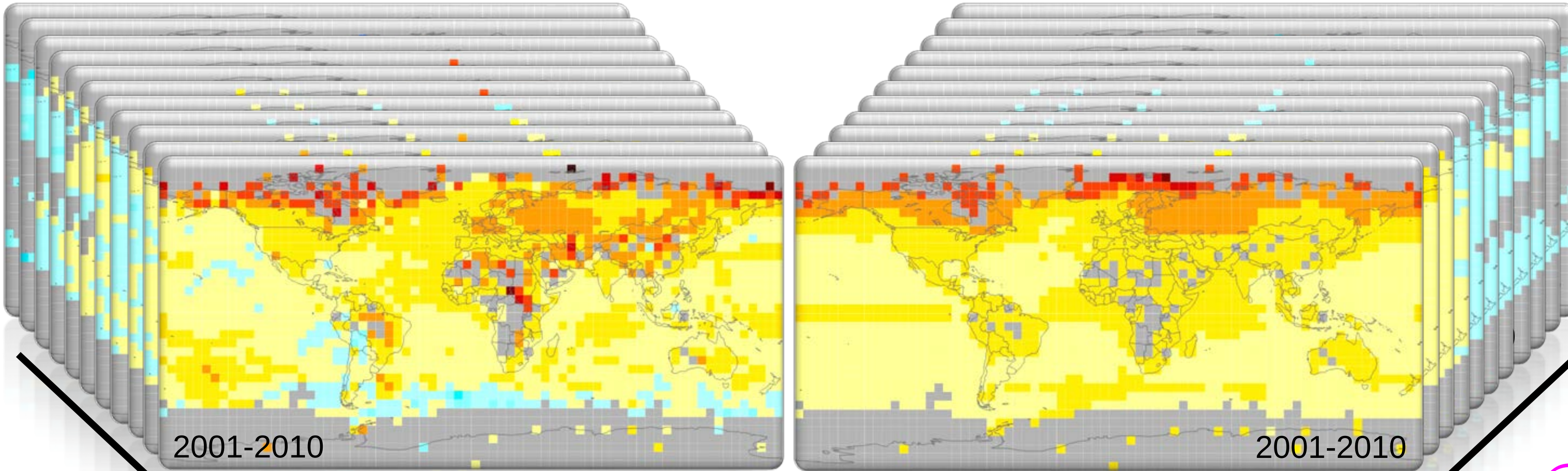
Usual assumptions

- Key forcings have been identified
- Signals and noise are additive
- The large-scale patterns of response are correctly simulated by climate models, but signal amplitude is uncertain

→ leads to a regression formulation

Observations (HadCRUT4)

Multi-model mean (ALL forcings)



11 decades (1901-1911 to 2001-2011)

$\mathbf{Y}$

$\mathbf{X}$

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Evaluate scaling factors ← $\hat{\boldsymbol{\beta}}$

$\hat{\boldsymbol{\varepsilon}}$ → Evaluate residuals

After Weaver and Zwiers (2000)

Detection and Attribution Paradigms

$$\mathbf{Y} = \sum_{i=1}^{s} \boxed{\beta_i} \mathbf{X}_i + \boldsymbol{\varepsilon}$$

$$\mathbf{Y} = \mathbf{Y}^* + \boldsymbol{\varepsilon}_{\boldsymbol{y}}$$

$$\mathbf{X}_{\boldsymbol{i}} = \mathbf{X}_{\boldsymbol{i}}^* + \boldsymbol{\varepsilon}_{\boldsymbol{x}_{\boldsymbol{i}}}$$

$$\mathbf{Y}^* = \sum_{i=1}^{s} \boxed{\beta_i} \mathbf{X}_i^*$$

$$\mathbf{Y} = \mathbf{Y}^* + \boldsymbol{\varepsilon}_{\boldsymbol{y}}$$

$$\mathbf{X}_{\boldsymbol{i}} = \boxed{\mathbf{X}_{\boldsymbol{i}}^*} + \boldsymbol{\varepsilon}_{\boldsymbol{x}_{\boldsymbol{i}}}$$

$$\mathbf{Y}^* = \sum_{i=1}^{s} \mathbf{X}_i^*$$

- Hasselmann (1979, 1993)
- Hegerl et al (1996, 1997)
- Tett et al (1999)
- Allan and Stott (2003)
- Huntingford et al (2006)
- Hegerl and Zwiers (2011)
- Ribes et al (2013a, 2013b)
- Hannart et al (2014)
- Hannart (2015)

- Ribes et al (2016)

Ordinary least squares regression

$$\mathbf{Y} = \sum_{i=1}^{s} \beta_i \mathbf{X}_i + \boldsymbol{\varepsilon} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$\mathbf{Y}$ → Observations

$\mathbf{X}$ → Expected changes – one vector for each “signal”

$\boldsymbol{\beta}$ → Regression coefficients – aka “scaling factors”

$\boldsymbol{\varepsilon}$ → Residuals – internal variability

Idea is to interpret the observations with a regression model, where physics is used to provide representations of expected changes due to external influences, statistics is used to demonstrate a good fit, and physics is used to interpret the fit and rule out other putative explanations

Key statistical questions relate to the β_i’s and residuals $\boldsymbol{\varepsilon}$

$$\mathbf{Y} = \sum_{i=1}^{s} \beta_i \mathbf{X}_i + \boldsymbol{\varepsilon} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Key assumptions

- Responses to forcings are additive
- Expected patterns of response in vectors $\mathbf{X}_i$ are correct
- Residuals ε_j, $j=1, \ldots, n$ are zero-mean
- … some more, discussed later

No assumptions about the "covariance structure" of the residuals

This is a "small sample" statistical inference problem (even if vector $\mathbf{Y}$ is big, covering essentially the globe and the entire instrumental period)

To fit, chose $\boldsymbol{\beta}$ to minimize $\left\|\mathbf{Y} - \mathbf{X}\widehat{\boldsymbol{\beta}}\right\|_{\boldsymbol{\Sigma}}^{\mathbf{2}}$

where $\|\mathbf{Z}\|_{\boldsymbol{\Sigma}}^{\mathbf{2}} = \mathbf{Z}^{T}\boldsymbol{\Sigma}^{-\mathbf{1}}\mathbf{Z}$

That is, we have a choice as to how we measure distance ….

$$\Sigma = \mathrm{I}$$

←Simple least squares, non-optimal

$$\Sigma = diag(\sigma_1^2, \ldots, \sigma_n^2)$$

←Weighted least squares, partially optimized

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1,n} \\ \vdots & \ddots & \vdots \\ \sigma_{n,1} & \cdots & \sigma_n^2 \end{pmatrix}$$

←Generalized linear regression, fully optimized

Minimizing $\left\|\mathbf{Y}-\mathbf{X}\widehat{\boldsymbol{\beta}}\right\|_{\boldsymbol{\Sigma}}^{2}$ yields

$$\widehat{\boldsymbol{\beta}} = (\mathbf{X}^t\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1}\mathbf{X}^t\boldsymbol{\Sigma}^{-1}\mathbf{Y}$$

Let $\boldsymbol{\Sigma} = \mathbf{P}\boldsymbol{\Lambda}\mathbf{P}^t$ where $\boldsymbol{\Lambda} = \boldsymbol{diag}(\boldsymbol{\lambda_1}, \ldots, \boldsymbol{\lambda_n})$

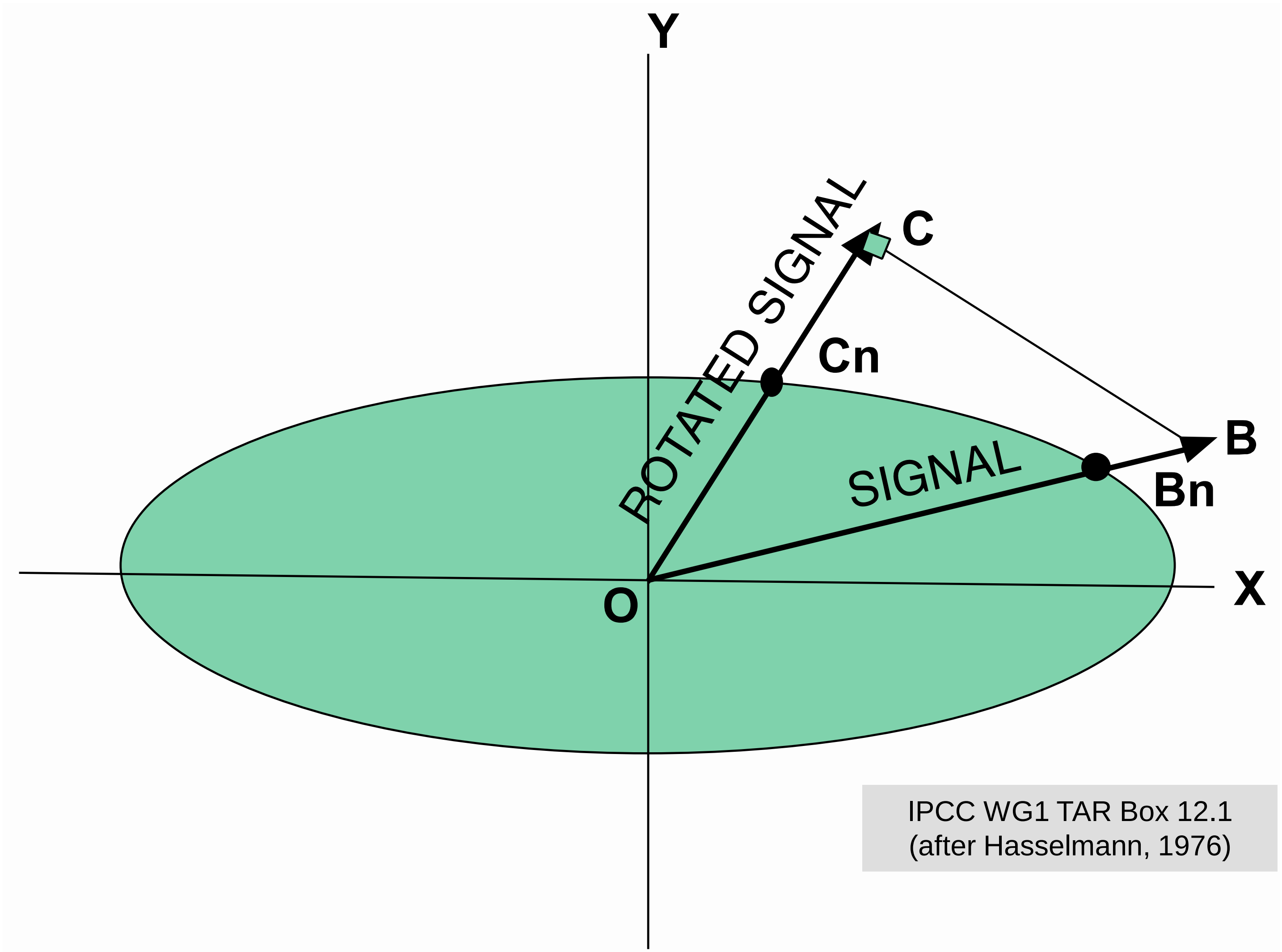
Then
$$\widehat{\boldsymbol{\beta}} = (\mathbf{X}^t\mathbf{P}\boldsymbol{\Lambda}^{-1}\mathbf{P}^t\mathbf{X})^{-1}\mathbf{X}^t\mathbf{P}\boldsymbol{\Lambda}^{-1}\mathbf{P}^t\mathbf{Y}$$
$$= (\acute{\mathbf{X}}^t\acute{\mathbf{X}})^{-1}\acute{\mathbf{X}}^t\acute{\mathbf{Y}}$$

Where
$$\acute{\mathbf{X}} = \boldsymbol{\Lambda}^{-1/2}\mathbf{P}^t\mathbf{X}$$
$$\acute{\mathbf{Y}} = \boldsymbol{\Lambda}^{-1/2}\mathbf{P}^t\mathbf{Y}$$

Thus the signals $\mathbf{X}$ and observations $\mathbf{Y}$ are being rotated and scaled

Optimization

- maximize S/N ratio by projecting observations onto the signal component that is least affected by noise



Applying the simple OLS form

Observations $\mathbf{Y}$

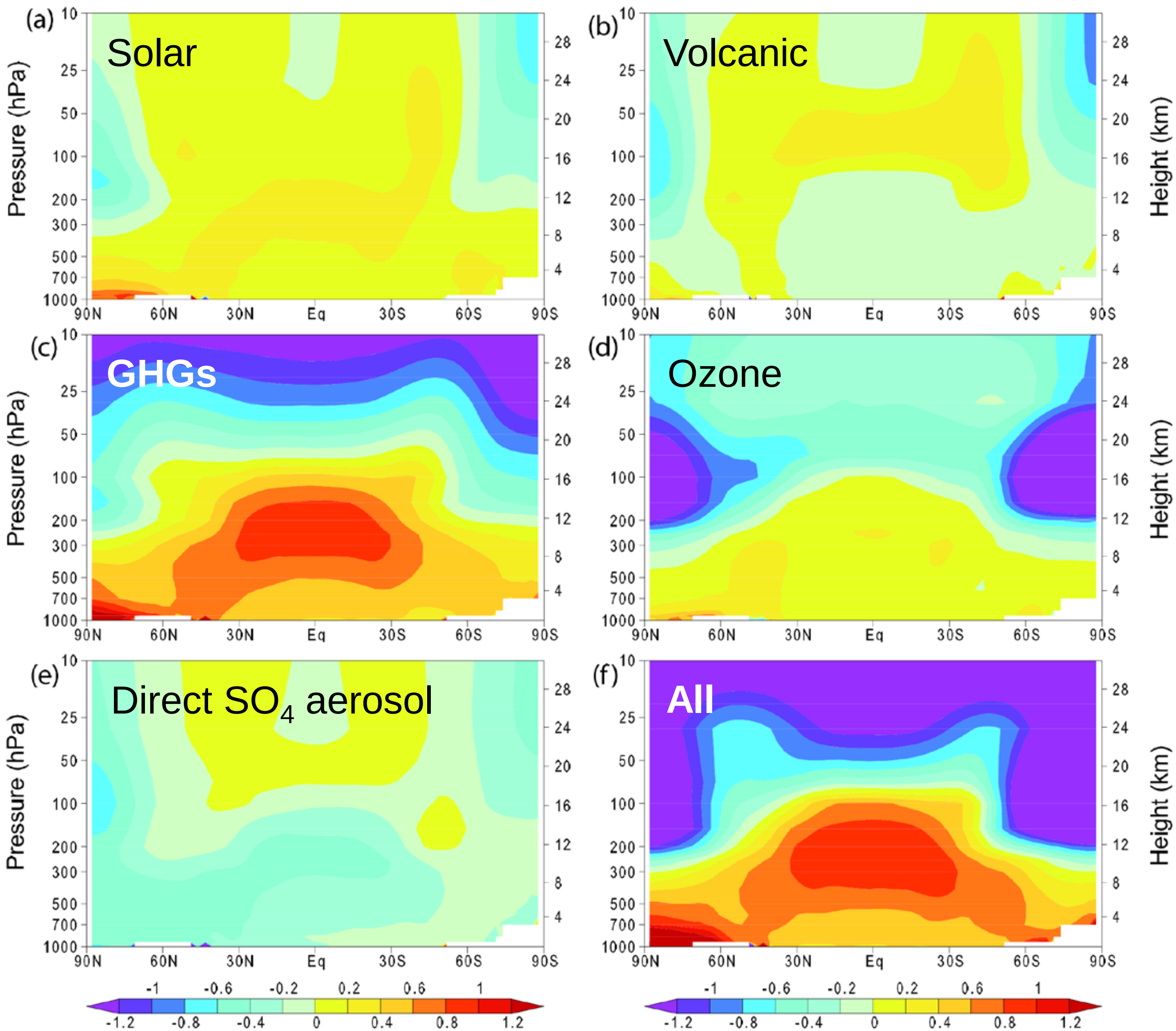
- Most studies of surface air temperature use
  - decadal averages and some kind of spatial averaging
    - To reduce noise from internal variability
    - To reduce the dimension of $\mathbf{Y}$
- Recent studies (e.g., Jones et al, 2013) use
  - Gridded (5°×5°) monthly mean surface temperature anomalies (e.g., HadCRUT4, Morice et al, 2012)
  - Reduced to decadal means for 1901-1920, 1911-1920 … 2001-2010 (11 decades)
  - Often spatially reduced using a "T4" spherical harmonic decomposition ⇒ global array of 5°×5°decadal anomalies reduced to 25 coefficients
  - $\mathbf{Y}_{n\times1}$ therefore has dimension $n=11\times25=275$

Signals X_i, i=1, …, s

- Number of signals s is small
  - s=1 → ALL
  - s=2 → ANT and NAT
  - s=3 → GHG, OANT and NAT
  - s=4 → …
- Can’t separate signals that are “co-linear”
- Signals estimated from either
  - single model ensembles (size 3-10 in CMIP5) or
  - multi-model ensembles (~172 ALL runs available in CMIP5 from 49 models, ~67 NAT runs from 21 models , ~54 GHG runs from 20 models)
- Process as we do the observations
  - Transferred to observational grid, “masked”, centered, averaged using same criteria, etc.

Examples of forced signals

PCM simulated 20th century temperature response to different kinds of forcing



IPCC WG1 AR4 Fig. 9.1

The generalized regression estimator of $\boldsymbol{\beta}$ is

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^t \boldsymbol{\Sigma}^{-1} \mathbf{X})^{-1} \mathbf{X}^t \boldsymbol{\Sigma}^{-1} \mathbf{Y}$$

Need an estimate $\hat{\boldsymbol{\Sigma}}$ of $\boldsymbol{\Sigma}$

- Usually estimated from control runs
- Even with decadal+T4 filtering, $\boldsymbol{\Sigma}$ is 275x275
  - need >275 110-year “chunks” of control run for a full-rank estimate

➔ Need further dimension reduction

- Constraints on dimensionality
  - Need to be able to invert covariance matrix $\hat{\boldsymbol{\Sigma}}$
  - Covariance needs to be well estimated
  - Climate model should represent internal variability well
  - Should be able to represent signal vector well

And one further constraint on estimating $\mathbf{\Sigma}$

– To avoid bias, optimization and uncertainty analysis should be performed separately (Hegerl et al, 1997)

➔ Require ***two*** independent estimates of of the covariance matrix

– An estimate $\widehat{\mathbf{\Sigma}}_{\mathbf{1}}$ for the optimization step and to estimate scaling factors $\boldsymbol{\beta}$

– An estimate $\widehat{\mathbf{\Sigma}}_{\mathbf{2}}$ to make estimate uncertainties and make inferences

- Residuals from the regression model, $\hat{\boldsymbol{\varepsilon}} = \mathbf{Y} - \mathbf{X}\widehat{\boldsymbol{\beta}}$ are used to assess misfit and evaluate model based estimates of internal variability

Total least squares

Do we really know the signal perfectly, and how do proceed if we don’t know it completely?

Statistical model for $\mathbf{X}_i$

– a single climate simulation j, $j=1,\ldots m_i$, for forcing i produces

$$\widetilde{\mathbf{X}}_{i,j} = \mathbf{X}_i + \boldsymbol{\delta}_{i,j}$$

Simulated 110 year change vector	=	Deterministic forced response	+	Internal variability

$$\Rightarrow \quad \widetilde{\mathbf{X}}_{i,\cdot} = \mathbf{X}_i + \boldsymbol{\delta}_{i,\cdot} \qquad \text{where} \quad \Sigma_{\delta\delta} = \frac{1}{m_i}\Sigma_{\varepsilon\varepsilon}$$

That is, we assume that the $\boldsymbol{\delta}_{i,j}$'s are independent, and that they represent repeated realizations of the internal variability $\boldsymbol{\varepsilon}$ of the observed system.

Leads to a more complicated regression model

$$\mathbf{Y} = \mathbf{Y}^{Forced} + \boldsymbol{\varepsilon}$$

$$\widetilde{\mathbf{X}} = \mathbf{X}^{Forced} + \boldsymbol{\Delta}$$

$$\mathbf{Y}^{Forced} = \mathbf{X}^{Forced}\boldsymbol{\beta}$$

Fitting involves finding the scaling factors $\boldsymbol{\beta}$ and signals $\mathbf{X}^{Forced}$ that minimize the “size” of the n×(s+1) matrix of residuals $[\boldsymbol{\Delta}, \boldsymbol{\varepsilon}]$

Covariance matrix estimation

Covariance matrix estimation

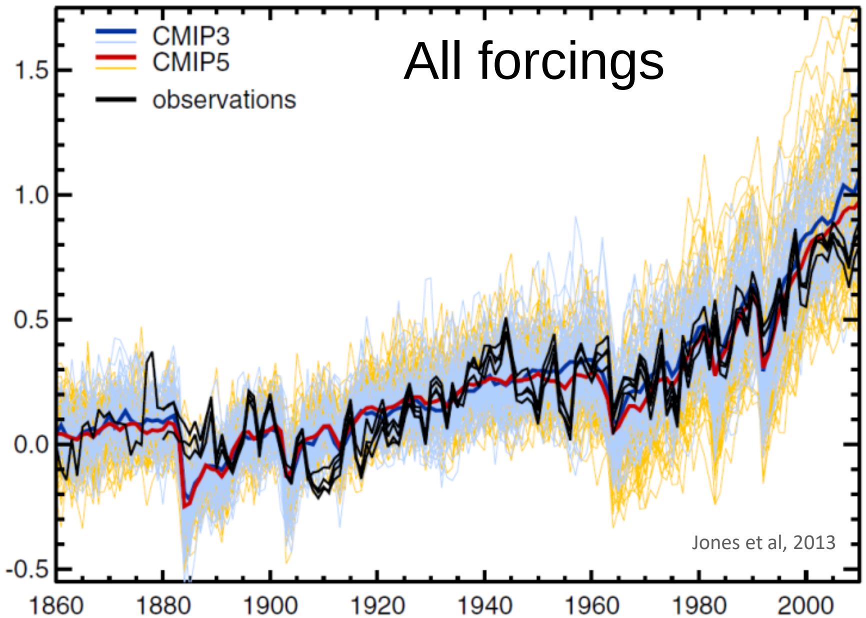
- "Even" with CMIP5, we often do not have enough information to estimate $\mathbf{\Sigma}$ well
- Keep dimensionality small (i.e., make the D&A problem only as complex as necessary)
  - Increases signal-to-noise ratio
  - Eliminates the need for EOF truncation
  - Forces explicit space- and time-filtering decisions prior to conducting the D&A analysis
  - Involves a trade off (e.g., we might lose the ability to distinguish between different signals)
- Use a "regularized" covariance matrix estimator

$$\widehat{\mathbf{\Sigma}} = \boldsymbol{\lambda}\widehat{\mathbf{C}} + \boldsymbol{\rho}\mathbf{I}$$

Discussion

Global warming attribution

Global mean temperature relative to 1880-1919



See also Figure 10.1, IPCC WG1 AR5

Trend in global surface temperature (1951-2010)

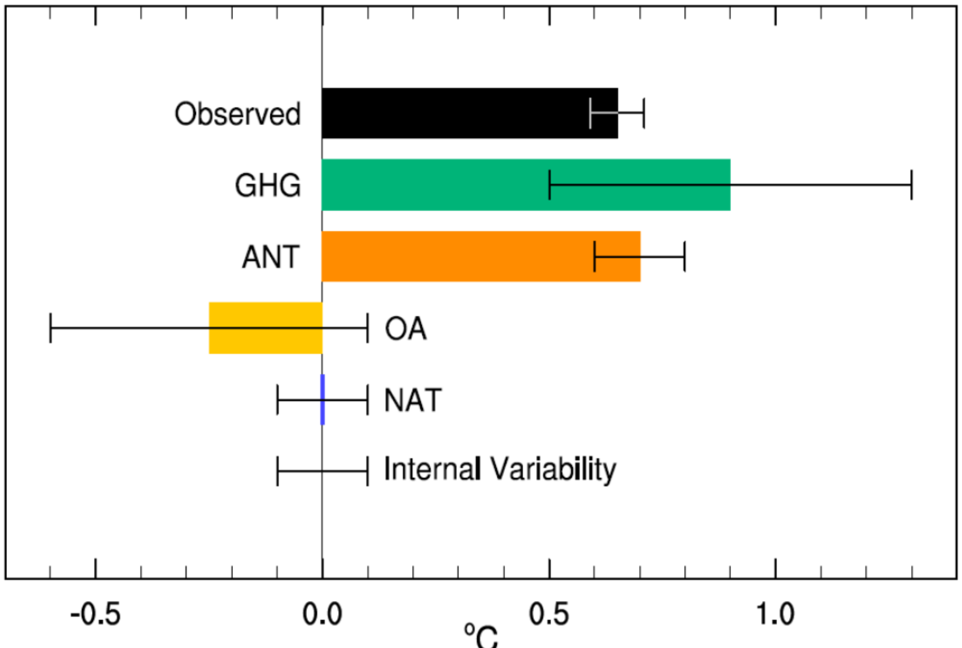


Figure TS.10, IPCC WG1 AR5

It is *extremely likely* that human influence has been the dominant cause of the observed warming since the mid-20th century.

“Trend” D&A Summary

- Quantifies how the mean state (or some other statistic) has changed over time due to forcing
- Methods are regression based, rely on additivity assumption, and continue to evolve
- Examples
  - Global and regional mean temperature
    - Large body of literature, very high confidence
  - Temperature extremes
    - Growing literature, high confidence
  - Precipitation extremes
    - Emerging evidence, medium or lower confidence

Some concerns

- Most studies implicitly assume Gaussian noise (generally not a large concern)
- Sampling variability in the estimated noise covariance matrix is not accounted for well in inferences
  - Hannart (2016) proposes a solution
- Most studies treat inter-model differences as sampling variability equivalent to internal variability
  - Hannart et al (2014) proposes a partial solution
  - Ribes et al (2016) propose an alternative approach
  - In reality, we do not have a comprehensive statistical framework that allows us to describe how the available ensembles of opportunity have been obtained

Some concerns …

- Many studies still use ad-hoc methods for covariance matrix regularization (e.g., EOF-truncation)
  - Some now use better approaches (e.g., the Ledoit-Wolf (2004) estimator) following Ribes et al (2013a, 2013b)

- Many studies do not discuss basic assumptions
  - Key forcings have been identified (and thus there are no other confounding influences)
  - Additivity of signals and noise, independence of noise on mean state
- Tendency to attribute based only on statistical evidence (see discussion in Mitchell et al., 2001)


Questions?
Photo: F. Zwiers